Modeling the Implied Volatility Surface:

An Empirical Study for S&P 500 Index Option

by

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Abstract

Volatilities play a critical role in financial industry as it is considered a common method to measure the risk. In order to minimize the risk or at least to be certain about how much risk is to be taken, investors seek various ways to estimate volatilities. The implied volatility surface is one of those methodologies and it displays the structure of volatilities as it varies with strike price (or moneyness) and time to maturity. The method can also be used to forecast the future volatilities. This paper aims, to present a simple framework to capture the common characteristics of this structure based on the S&P 500 options in real time, to address the issue of the thin volume of option trading and missing data, and to present some possible applications of the methodology. The parametric model of Dumas, Fleming and Whaley (1998) is used in this paper to estimate the implied volatility surfaces for given option moneyness and time to expiration.

Key words: implied volatility surface, thinly traded, moneyness

Acknowledgement

The writing of this thesis is a journey filled with obstacles and learning. It is a bona fide fortune for my partner and me that we have so many people around us from whom we've gained support, advice, and inspiration. If not for all of them, there would never be a thesis on modeling the implied volatility surface.

We first need to thank Dr. Jussi Keppo, for his inspiration on us to do this project. Actually, my partner and I formed the idea of modeling the implied volatility surface after we did the first assignment for Dr. Jussi Keppo's class. His detailed and insightful feedback on our assignment really shed some light on us.

Before starting doing the final project, my partner and I were a little bit confused and unsure of which sub-theme we should choose for our final project because there were two ideas going on at that time. And merely ideas would never complete a good thesis. For that, we're sincerely grateful to our supervisor, Dr. Christina Atanasova. Her guidance and patience cleared up the clouds. Her knowledge of volatility modeling, with her constructive suggestions, has been a great boon to us. Owing to her, we could finish this paper in a good shape and right on time.

And personally, I am in debt to one of my friends most regarding this project. Edward Lee, who has brainstormed with me on various topics on implied volatility, is one of the drives for which I can achieve this success with my partner. I really appreciate all the things that he has done for us.

For those who we haven't mentioned, please accept our apologies. Our words may seem short, but our gratitude is long and deep.

Aldaris,

together with Chenguang

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Section I: Introduction

In the basic classic Black and Scholes (1973) model, option prices are derived from the following five variables:

- The current market price of the underlying asset,
- The strike price for the option,
- The risk-free interest rate,
- The volatility of the underlying asset,
- And the option's time to maturity.

All of the above variables are observable from the option contract and the market trading, except for the volatility. Since the prices of the options are also available from the market, a standard way to calculate the volatility from the BS model is to solve for it given an option price. By plugging in all the observable variables to the BS model, the annual volatility for the underlying asset could be resolved, namely the implied volatility. BS model assumes that volatility is constant; nonetheless, different volatilities can be obtained based on different strike prices or distinctive times to expiration via the same fashion. An implied volatility surface could then be formed by plotting all the implied volatilities in terms of strike prices (or moneyness) and time to maturities.

In the classic BS setting, all options with the same expiration on the same underlying asset prices should yield the same volatility. In other words, the implied volatility surface should be flat if plotted. However, this differs from the actual results gained from using the market prices. It is well known that the volatilities over different strikes exhibit the shape of a smile; that is at-the-money options derive the lowest volatilities, while as the option goes out-of-the-money or in-the-money, the implied volatilities increase (Cont and Fontseca, 2001). From the perspective of term structure, the implied volatilities of long-term options exceed that of the short-term options (Derman, Kani and Zou, 1995). Some practitioners might argue that this so-called implied volatility might not be reliable to use since there is the likelihood that implied volatilities on some particular dates are determined randomly. In order to obtain realistic implied volatilities, a model is needed to smooth out the raw implied volatilities computed from the discrete data. Derman & Kani (1994) have developed a model where the volatility is presented as a deterministic function of the asset price and time to maturity. Ncube (1996) used a regression model to estimate the volatility of FTSE 100 based on strike price and time to expiration. Also, Dumas, Fleming and Whaley (1998) proposed some parametric models to fit the implied volatilities for S&P 500. Later, Peňa, Serna and Rubio (1999) estimated the implied volatilities for the Spanish index as function of moneyness using six different structural forms. More recently, Alentorn (2004) modified Dumas, Fleming and Whaley's (1998) models and estimated the implied volatilities for FTSE 100 with a different time horizon on moneyness and time to maturity. And in this paper, Alentorn's (2004) models would be examined on the latest S&P 500 data, taking into account the issue of thinly trading.

The remainder of the paper is organized as follows. Section II will introduce the methodology for calculating the implied volatility and parametric models that are utilized in this paper based on Alentorn's (2004) work. After a description of the data and research methodology in Section III, the statistical results of the model and the implied volatility surfaces will be illustrated in Section IV. Then in Section V, the problem of thinly trading and missing data will be discussed, followed by Section VI, which presents a re-visit of the model after the adjustments made dealing with the prior problem. Afterwards, some applications of the implied volatility will be introduced in Section VII. And eventually, the conclusions, together with some of the limitations when doing the project, will be presented in Section VIII.

Section II:

1. Calculation of the implied volatility

A common method to compute the volatility for a given underlying asset is to invert an option pricing formula given the market price of the option. The option price is obtainable from the market. By applying a suitable option pricing formula, some certain volatility could be easily computed in a backward fashion. In this manner, the volatility is implied by the option price; thus it is well known as the implied volatility. In this paper, the classic Black and Scholes model is adopted to calculate the implied volatility.

Recall the Black and Scholes formula for a European call option:

$$C = S_0 e^{-\delta T} N(d_1) - K e^{-rT} N(d_2)$$

And for a European put option:

$$P = K e^{-rT} N(-d_2) - S_0 e^{-\delta T} N(-d_1)$$

where S_0 is current price of the underlying asset, δ is the continuously compounded dividend yield, K is the option's strike price, T is the option's the time to maturity, r is the risk-free interest rate. And

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

where σ is the volatility that is to be solved in this paper, and $N(d)$ is the standard normal cumulative distribution. The implied volatility could be resolved uniquely, via either of the corresponding equations, from each price of any traded option owing to the monotonicity of the Black and Scholes formula (both for call and put) with respect to σ, that is

$$\frac{dBS}{d\sigma} > 0$$

2. The dividend yield and the risk-free interest rate

Theoretically, the dividend yields and the risk-free interest rates need to be synchronized for the duration of the option. δ is the dividend yield rate for a certain asset (or an index) during the life time of a future contract that has the same expiration date as the option. The equation for that is as follows:

$$F_0 = S_0 e^{(r-\delta)T}$$

Moreover, the put-call parity shows that:

$$C + Ke^{-rT} = P + S_0 e^{-\delta T}$$

Hence, combining the above two equations, a timely-fitted dividend yield δ and risk-free interest rate could be retrieved as long as the rest of the variables are obtainable, which are:

- The price of the underlying asset, denoted as S_0
- The price of a call option, denoted as C
- The price of a put option with the same strike price and time to maturity as the call, denoted as P
- The strike price, denoted as K
- The value of the future contract with the same expiration date as the two options, denoted as F_0

However, the methodology above is not easy to implement. For one, usually only a relatively small portion of the options with the same underlying asset are actively traded (the options close to at-the-money, and close to expiration); even index options, which are the most liquid, have the same issue. For another, it is rather difficult to find a future contract with the same maturity as the option. Therefore, in this paper the dividend yield rate and the risk-free rate are not derived via the theoretical way described above. Instead, market data are used and these two parameters are directly taken from the Bloomberg database.

3. The construction of the smooth surface

As mentioned Section I, several studies have introduced models to fit the implied volatilities. Dumas, Fleming and Whaley (1998) estimated the implied volatilities over two dimensions: strike level and time to maturity. Taking into the consideration that Cont (2002) has argued that implied volatilities are better exhibited across moneyness than across strike, Alentorn (2004) tested the models in Dumas, Fleming and Whaley's paper (1998), but exchanged the strike parameter with moneyness. On top of that, this also adds to the benefits for computation because running regression based on moneyness instead of strike is faster.

This paper will examine the following three models:

Model 0: $\sigma(M, \tau) = \alpha + \varepsilon$

Model 1: $\sigma(M, \tau) = \alpha + \beta_1 M + \beta_2 M^2 + \varepsilon$

Model 2: $\sigma(M, \tau) = \alpha + \beta_1 M + \beta_2 M^2 + \beta_3 \tau + \beta_4 \tau * M + \varepsilon$

Model 0 would present a constant volatility regardless of strike (or moneyness) and time to maturity, which coincides with the BS model's assumption. It doesn't have much practical value and only serves as a means of comparison in this paper.

Model 1 incorporates both the linear and quadratic relations between moneyness and volatility. A better statistical result is expected via model 1 than model 0.

Model 2 assumes that volatility is affected by several factors. Firstly, there is a constant factor α contributing to the volatility regardless of other factors. This somewhat coincide with the BS model which assumes a constant volatility. Secondly, moneyness also plays a role, in both linear and quadratic terms. The third assumption is that volatility varies through time. And last but not least, time affects the volatility in synergy with moneyness. For the term moneyness, it could be defined in many different ways, such as K/S, S/K, $\ln(S/K)$ and $\frac{\ln(S/K)}{\sqrt{T}}$, etc. In Gross and Waltners's (1995) paper, moneyness is defined as $\frac{\ln(S/K)}{\sqrt{T}}$. However, $M = \ln(S/K)$ is adopted in this paper, for the rationale that there won't be time effect in the moneyness term.

All the αs in the models are the constant terms of the regression. As mentioned previously, these αs preserve the basic assumption of the BS model that volatility is constant, to some extent. The well-known volatility smile is captured by a quadratic term in model 1 and model 2. β_1 is the location coefficient which determines the displacement of the "smile". β_2 is the shape coefficient which controls how the "smile" looks like. β_3 exhibits the term structure of the implied volatility. And β_4 presents the combined effects of time and moneyness on implied volatility.

In theory, call and put options with the same underlying assets should yield the same implied volatility. However, Ncube (1996) argues that volatilities implied via put options might be higher than those derived from call options because investors would be willing to pay more for the nature of the put option that it is a good instrument for hedging. But in this paper, this possible bias would not be taken into account as the above phenomenon is not found among the implied volatilities computed from the selected data. (see the next section)

Section III:

1. Data analysis

The data sets used in this paper are downloaded from the Bloomberg database. Originally, a three-month range of option prices on S&P 500 has been retrieved from the database as well as the performance of the index itself within the same time period. Due to the unavailability of some of the data and the inactiveness of some of the options during a certain time interval, the data from July 25th to July 29th, 2011 are utilized. The implied volatility could be resolved on a daily basis because each option price is timestamped and could be timely matched with the underlying index. And the prices for all the options are the last price of the day, though some of the at-the-end-of-day prices may be actually from sometime earlier during the day due to the fact that not all the options were continuously traded. On average, there were approximately 3000 contracts traded every day for each of the options with the selected strike prices and time to maturities, and those which are at-the-money and close to maturity tend to have higher volumes.

Table 3.1 and Table 3.2 display the implied volatilities computed from calls and puts respectively as of July 25th, 2011.

Strike	Maturity					
	20/08/2011	17/09/2011	22/10/2011	17/12/2011	17/03/2012	16/06/2012
1250	0.2410	N/A	N/A	0.1894	N/A	N/A
1275	0.1876	0.1521	N/A	N/A	N/A	N/A
1300	0.1645	0.1618	N/A	0.1751	N/A	N/A
1325	0.1609	0.1505	0.1686	0.1590	N/A	0.1694
1350	0.1344	0.1356	0.1461	0.1515	0.1594	0.1647
1375	0.1254	0.1289	0.1379	0.1421	N/A	N/A

Table 3.1

Strike	Maturity					
	20/08/2011	17/09/2011	22/10/2011	17/12/2011	17/03/2012	16/06/2012
1250	0.1863	0.1738	0.1720	0.1695	0.1691	N/A
1275	0.1719	0.1607	0.1596	0.1644	N/A	N/A
1300	0.1549	0.1507	0.1521	0.1519	N/A	0.1617
1325	0.1389	0.1398	0.1434	0.1454	N/A	0.1585
1350	0.1289	0.1267	0.1327	0.1388	0.1459	0.1509
1375	0.0950	0.1146	0.1205	N/A	N/A	N/A

Table 3.2

As shown above, the implied volatilities calculated from call options are not necessarily lower than those calculated via put options in practice. Volatilities via call options average at 0.1594 while volatilities via put options have an average of 0.1492. This somehow bellies the argument made by Ncube (1996).

2. Regression procedure

The Ordinary Least-Squared (OLS) method, which is commonly used for regression, is applied in this paper. The OLS method measures the deviations of the estimated results from the actual observations and gives out all the coefficients (βs) and the intercept (α) by minimizing the sum of squared errors (SSE). Those error terms are the differences between the actual observations y_i and the estimated numbers $\hat{y}_i$. The process could be expressed as below:

$$min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

A standard measure of how well the estimation fits the actual data is R^2. The larger the R^2 is, the better the independents could explain the dependents. The R^2 is calculated as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

In addition, people would look at the t-stats for all the coefficients to see the significance of the explaining terms. It is calculated as follows:

$$t = \frac{\hat{\beta}}{s.e.(\hat{\beta})}$$

where $\hat{\beta}$ is estimated coefficients for the parameters and $s.e.(\hat{\beta})$ is the standard error of those coefficients.

Section IV: Statistical results and the estimated implied volatility surface

All the three models have been tested for five selected trading days from the downloaded data sets. The results are shown in the following tables and graphs. In the table, the figures in parentheses are the t-stats for all the coefficients, including the intercept. The red-colored numbers are the critical values for t-test with a 95% confidence interval.

Table 4.0 below displays the estimated αs (the constant parameter) for Model 0, which has a R^2 of zero since it is a constant function.The average implied volatility is 0.1876. And Figure 4.0 shows the surface of the estimated implied volatilities for July 29th, 2011, with the blue dots representing the original observations. Not surprisingly, the surface is flat, presenting the basic assumption of the BS model.

Date	Parameters (t-stat)	R squared
	α	t-critical
July 25, 2011	0.1789	0.0000
	(50.8383)	2.0117
July 26, 2011	0.1844	0.0000
	(54.3222)	2.0086
July 27, 2011	0.1887	0.0000
	(54.5269)	2.0066
July 28, 2011	0.1923	0.0000
	(58.5381)	2.0106
July 29, 2011	0.1938	0.0000
	(60.6804)	2.0049

Table 4.0

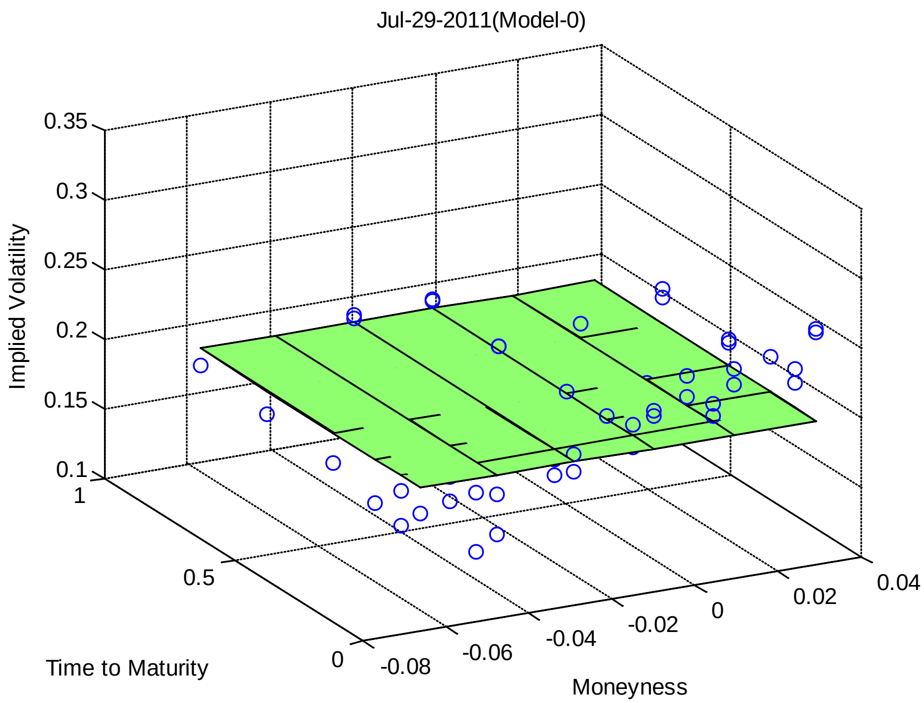

Figure 4.0

The regression results for Model 1 are exhibited below in Table 4.1.

The R^2s proliferate drastically (average at 0.6762) after adding the moneyness terms, both linearly and quadratically.

The αs for the first two days decrease while those for the rest of the week increase compared to Model 0. The average of the constant terms is slightly reduced to 0.1857, indicating that part of the implied volatility might be explained by moneyness.

For βs, β_2s are larger than β_1s in absolute value, suggesting volatility and moneyness are better related in a quadratic fashion than linear. However, all β_2s fail the t-test under a 95% confidence interval, implying that the quadratic moneyness terms are not statistically significant. Figure 4.1 shows the new surface generated via Model 1. It presents a not-so-obvious parabolic surface for the implied volatilities across moneyness with no term structures.

Date	Parameters (t-stat)			R squared
	α	β_1	β_2	t-critical
July 25, 2011	0.1691	0.6843	-0.8830	0.6607
	(62.2778)	(5.7849)	(-0.3675)	2.0117
July 26, 2011	0.1751	0.6182	1.1643	0.7130
	(69.1912)	(6.9333)	(0.5593)	2.0086
July 27, 2011	0.1899	0.6880	2.7221	0.6881
	(65.1221)	(10.5768)	(1.2542)	2.0066
July 28, 2011	0.1935	0.6594	2.4910	0.6878
	(73.2360)	(9.9185)	(1.2073)	2.0106
July 29, 2011	0.2010	0.6756	2.3234	0.6316
	(72.1735)	(7.7126)	(1.0372)	2.0049

Table 4.1

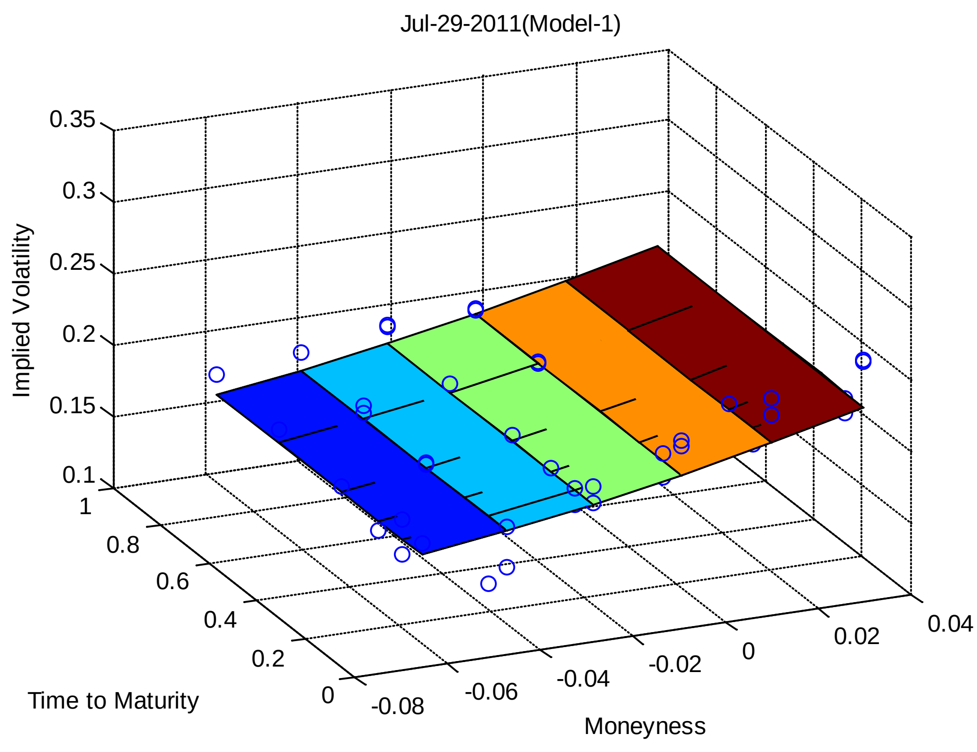

Figure 4.1

Lastly, Table 4.2 shows the estimated results for Model 2 and Figure 4.2 is the corresponding implied volatility surface. Apparently, having two more explaining terms for the model increase the R^2s significantly (averaging at 0.8005), as a result of capturing the term structures of the implied volatilities.

Additionally, the average of αs diminish (to 0.1808) again due to the increased explaining power of the βs. β_2 and β_4 are noticeably larger than β_1 and β_3 respectively (except for July 26th, 2011 where β_2 is smaller than β_1), again implying that volatility could be better explained by quadratic terms. The same issue lies here as well that all the β_2s fail the t-test under a 95% confidence interval, which may arouse questions on quadratic moneyness terms. β_3s are relatively small than any of the other coefficients, nevertheless all of them pass the t-test. In other words, time does play a role in determining the volatility, but only with a minor impact. Another fact that contradicts former studies is that the sign of β_4 is negative, and the t-stats are all greater than the critical values on the five dates. This could be interpreted that time combined with moneyness has a negative effect on volatility.

And from the graph, the smile over moneyness is more apparent that on the graph corresponding to Model 1. In addition, volatility does go up as time evolves.

Date	Parameters (t-stat)					R squared
	α	β_1	β_2	β_3	β_4	t-critical
July 25, 2011	0.1548	0.9020	1.5167	0.0401	-1.0771	0.8210
	(49.7744)	(8.5976)	(0.8318)	(6.0119)	(-4.3347)	2.0117
July 26, 2011	0.1633	0.9319	0.8901	0.0306	-0.7922	0.8907
	(74.1933)	(12.9880)	(0.6786)	(7.5484)	(-6.5741)	2.0086
July 27, 2011	0.1861	0.9811	1.5619	0.0117	-0.7590	0.8114
	(62.1883)	(12.2295)	(0.9007)	(2.1690)	(-4.5251)	2.0066

July 28, 2011	0.1941 (62.8311)	0.9096 (10.6851)	1.6384 (0.9036)	-0.0004 (-0.0671)	-0.7482 (-3.9783)	0.7720 2.0106
July 29, 2011	0.2055 (55.0460)	0.9408 (8.7531)	1.7751 (0.8675)	-0.0148 (-1.8454)	-0.8545 (-3.6608)	0.7075 2.0049

J Table 4.2 2)

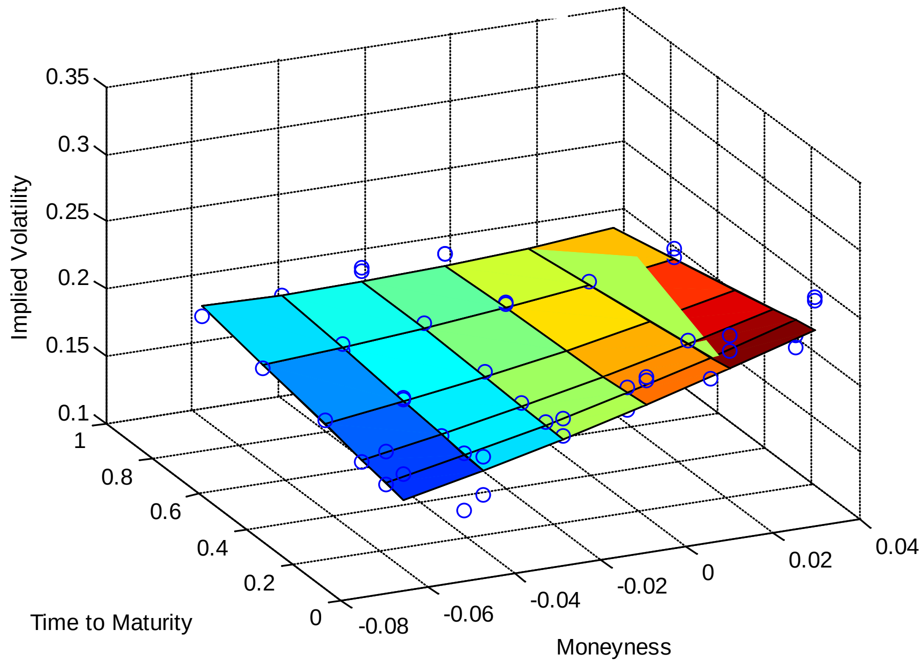

Figure 4.2

Section V: Dealing with thinly traded and missing data

Many of the S&P 500 option price datasets have missing data or are thinly traded, which causes the problem that the implied volatility surface might be rigged and inaccurate. For instance, deep in-the-money or deep out-of-the-money options have unreliable pricing because of the lack of liquidity, causing the options to be thinly traded or not traded at all. And as a matter of fact, missing data, which indicates that there is no trade on this particular option on a particular date, is just the extreme case of thinly trading. To address this problem, a series of approaches is adopted to modify the data. Ideally, it would be better if a threshold could be set up to determine whether an option is thinly traded or not. Nevertheless, in practice of the project, there is the scenario that both call and put are actively traded (both above a chosen threshold) or that both of them are not actively traded (both under a chosen threshold). Therefore, the notion of relatively thinly traded is introduced; that is the option which has the higher volume than its counterpart would be utilized to derive the implied volatility. For example, for the same date and the same strike price, the call option has a volume of 100 and the put option has a volume of 80; then the implied volatility would be computed via the price of the call option. This notion goes in line with the intuition that the more actively traded the option is, the more accurate the option price reflects its true value. And if this intuition is correct in real practice, then the statistical results would be improved after a re-visit of the regression models applying the relatively non-thinly traded option prices.

Overall, a three-step approach is taken to deal with the problem of thinly traded and missing data. The first step is described in the prior paragraph with the notion of relatively thinly traded. Then if both of the option prices on the same date with the same strike are not available, the implied volatility of the option from the adjacent dates would be replicated. For instance, the call option strikes at 1350 with expiration of March 17th, 2012 is not traded on July 27th, 2011, and the price of the counterparty put is not available as well; then the price of the 1350 call option expiration on the same date traded on July 26th would be adopted. Finally, if all these data are not obtainable, the intrinsic value of the option might be used.

Taking step one only would be ideal to perform the regression as choosing the data point with a higher volume would be beneficial in theory. Taking more steps would be likely to degrade the output of the regression. As for step two, using the volatility on another date (even if it is adjacent) would take out the time effect. And for step three, utilizing the intrinsic value is not that reliable because the intrinsic value for a particular date could be possibly determined by only two parties and with a very low volume.

Section VI: Re-visit of Model 2 and the modified implied volatility surface

As described in the prior section, the adjusted data are plugged into Model 2 again, in the hope of improving the statistical results. The after-modification results are illustrated in Table 6.1. As exhibited, the overall fitness of the data is raised, resulting in an average R^2 of 0.8864. Therefore, the approaches discussed in Section V are capable of improving the estimated results in practice.

As July 25th, 2011 is the most actively traded date, calls and puts are both listed almost on every strike with different time to maturities. Thus, there is usually the choice between the implied volatilities derived via call and via put, and the better one could always be selected to reflect the true market volatility. On contrast, there are many missing data points for July 29th, 2011, which makes it evitable to take step two described previously or for some cases step three. As expected, the R^2 is much smaller than that of July 25th, 2011. Furthermore, the R^2 for July 25th, 2011 now exceeds that for July 26th, 2011 due the same reason above. Before introducing the notion of relatively thinly traded, it is the opposite. Hence, it is another piece of evidence that the proposed approaches can make the regression results better off.

Figure 6.1 displays the modified implied volatility surface as of July 29th, 2011. The surfaces for the other four dates could be found in the appendix. Again, there is no clear sign for the “smile”.

Date	Parameters (t-stat)					R squared
	α	β_1	β_2	β_3	β_4	t-critical
July 25, 2011	0.1557	0.6585	1.8131	0.0392	-0.7000	0.9747
	(121.0600)	(15.7160)	(2.5855)	(14.1970)	(-7.6876)	2.0555
July 26, 2011	0.1619	0.7925	0.5431	0.0341	-0.6068	0.9362
	(76.1890)	(12.3020)	(0.4606)	(8.7954)	(-5.7356)	2.0555
July 27, 2011	0.1878	0.7826	0.6963	0.0088	-0.5766	0.8929
	(74.3850)	(12.0810)	(0.5093)	(2.0787)	(-4.5452)	2.0555
July 28, 2011	0.1945	0.7428	0.0023	0.0016	-0.6106	0.8750
	(73.7630)	(10.5730)	(0.0016)	(0.3147)	(-4.1588)	2.0555
July 29, 2011	0.2085	0.8136	0.5700	-0.0193	-0.7677	0.7531
	(48.6580)	(7.0261)	(0.2639)	(-2.1242)	(-3.2041)	2.0555

Table 6.1

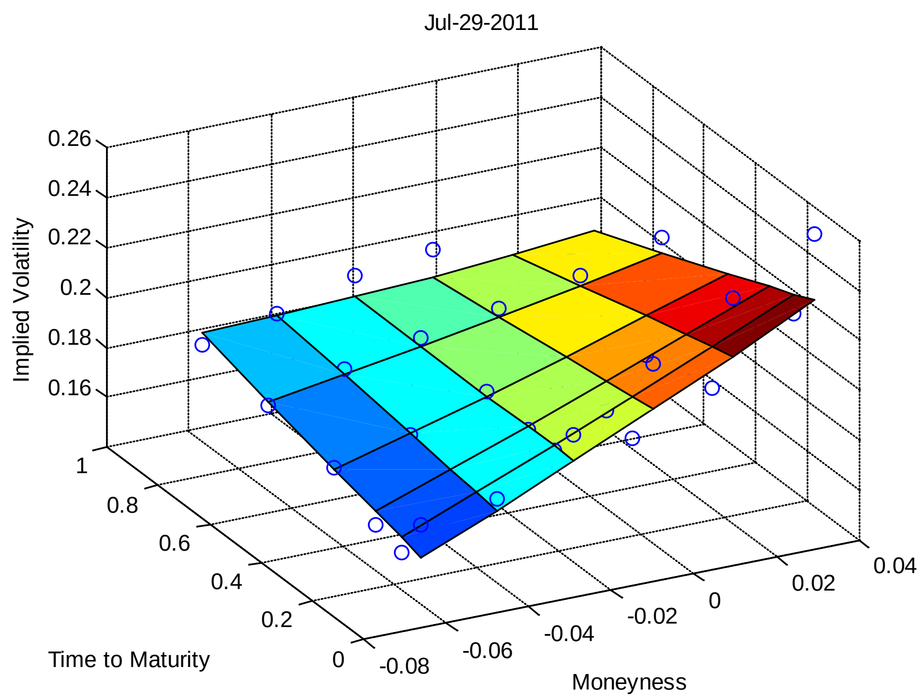

Figure 6.1

Section VII: Applications

There are many ways that the implied volatility could be applied in real practice of finance. For instance, the implied volatility is often used to price other illiquid options on the same underlying asset.

In 1994, Derman and Kani constructed an implied binomial tree based on the implied volatility surface in order to price path-dependent options. From an implied volatility surface, people can get the option prices for all strikes and time to maturities. Hence, the probability of reaching each node could be determined when constructing the implied binomial tree. This model enables practitioners to price path-dependent options consistently with observable vanilla option prices, yet has the flaw of negative probabilities. Later in 1996, Derman and Kani, together with Chriss, introduced another model to correct this problem by assigning an appropriate state space. Both models require an accurate, thus reliable, implied volatility surface that reflects the true option values to price those path-dependent options.

In 1995, Derman, Kani and Zou derived another volatility surface, namely the local volatility surface, from the traditional implied volatility surface. This framework abandons the notion of constant future volatility and extracts the market's consensus for future local volatilities as a function of future price of the underlying asset and time. This local volatility surface could be applied to form the implied binomial tree (in turn, to price path-dependent options), to estimate the future evolution of the "smile", and to price barrier options.

Besides being a tool in exotic option pricing and deriving local volatility surface, the implied volatility surface is also utilized in risk management to calculate the Value-at-Risk (VaR). VaR is a measure of the risk of maximum loss for a portfolio over a certain time horizon given a certain confidence interval. The estimated implied volatilities are forecasts of average future volatilities (Cassese and Guidolin, 2003); therefore those implied volatilities could be used to compute VaRs for portfolios containing any combination of the underlying assets and other derivatives with the same underlying assets.

Section VIII:

1. Conclusions

In this paper, three models have been tested on raw S&P 500 data, and additionally one of the models has been further examined on a modified data base. As expected, Model 2 fits the data best, which incorporates the term structure across time as well as the synergy of time and moneyness. Furthermore, Model 2 is better fitted after some adjustment of the data has been made which deals with problem of thinly trading. However, there is no obvious presence of the well-known volatility smile on the five tested days.

Some real-time applications are discussed in the previous section. Research on similar topics (pricing exotic option via implied volatility; calculating value-at-risk; etc) could be expanded in more quantitative forms in the future.

2. Limitations and future work

Data selection is one of the obstacles encountered while doing the project. The original data are collected in a three-month time frame, from April 1st, 2011 to July 29th, 2011. It is very common to observe that only a small portion of options are non-thinly traded, with those at-the-money and close to expiration being most actively traded. The U.S. derivative market is relatively matured and has a quite high volume of trade, yet there are still many unavailable data points in the original data set selected on S&P 500 options. In order to have more data for the sake of regression, but meanwhile to leave some room to address the problem of thinly trading, a subset of five consecutive trading days (from July 25th, 2011 to July 29th, 2011) are chosen to perform the framework presented in this paper. However, the S&P 500 index itself fluctuated quite widely during these five days (fell from1337.43 to 1292.28), making it difficult to pick the strike levels to form a symmetric axis for moneyness. Moreover, this could also be the cause for the absence of the volatility smile and some of the not-so-ideal statistical results, such as the insignificance of the time to expiration term. On top of that, further research might be needed to address the problem of negative sign for the combined time and moneyness term.

Another limitation lies in the procedure of regression, which is even more difficult to overcome. As the traditional OLS method minimizes the sum of errors squared, it treats all the error terms equally. One suggestion has been brought up by Alentorn (2004) that a weighted sum of errors squared could be minimized instead, to obtain a more accurate estimation while taking into account the time value of volatility. The weights would be inversely proportional of the time elapsed from the last observation, so that the more recent and relevant data would be assigned with more weights while the older data would have a slightly smaller impact. The process could be express as follows:

$$min \sum_{i=1}^{n} \frac{1}{(1+\tau_i)} (y_i - \hat{y}_i)^2$$

where τ_i is the time elapsed since option *i* starts to be traded.

Furthermore, the implied volatility in this paper is derived in a BS setting, which now is considered not a very sophisticated methodology. Hence, models such as Heston's

(1993) stochastic volatility model could be applied in the future to form a more precise volatility surface.

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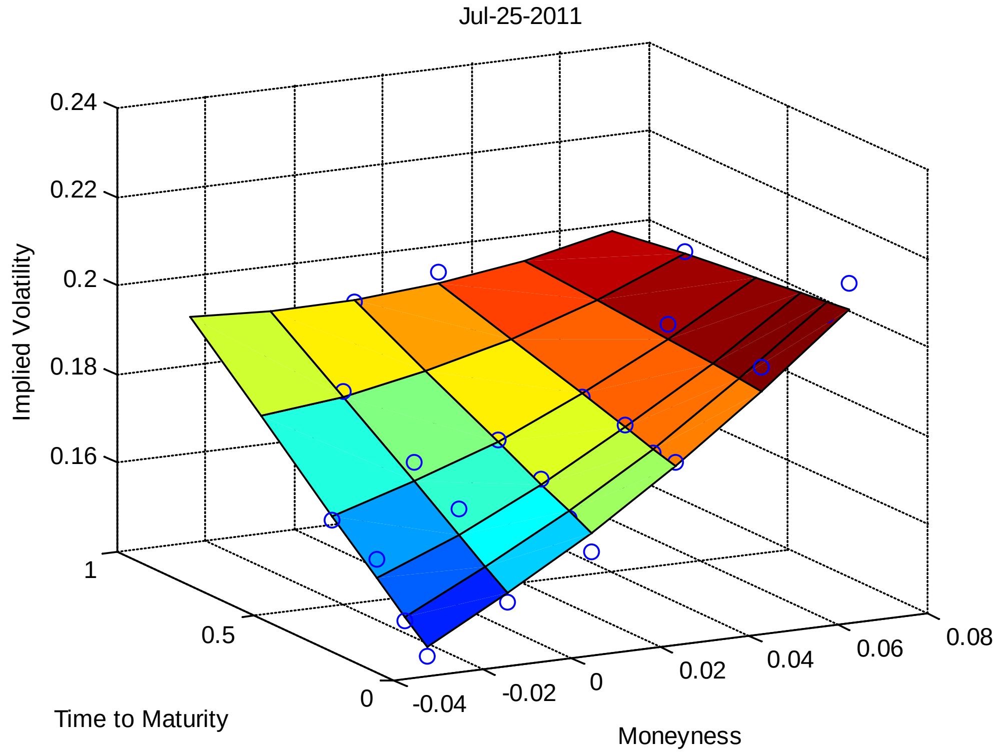
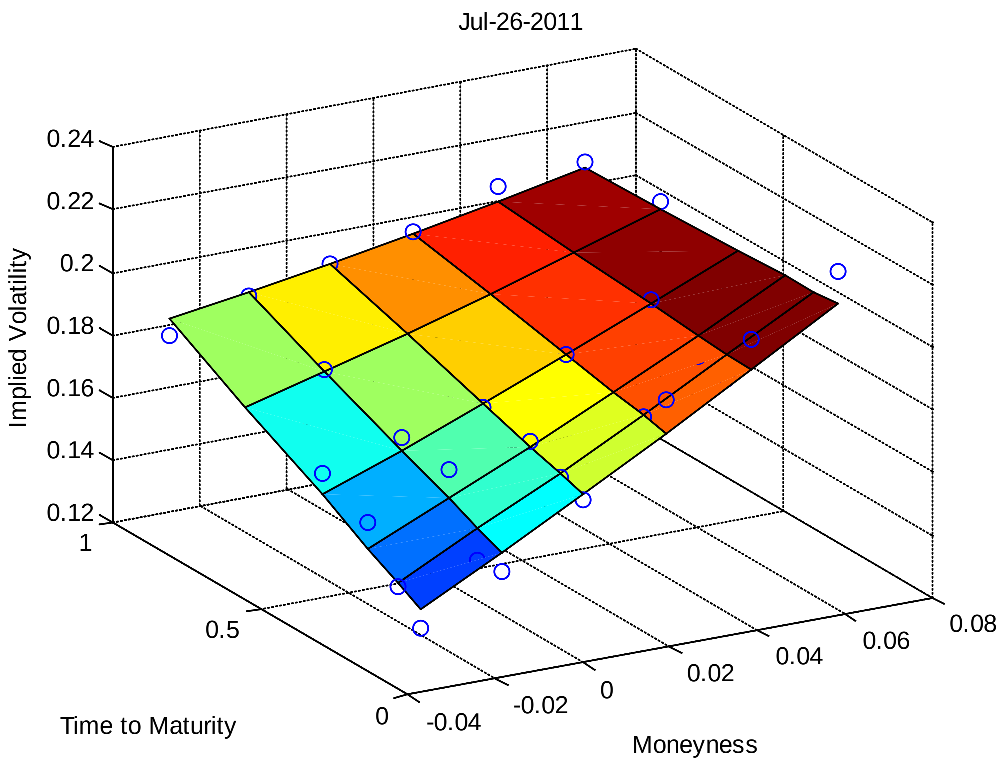
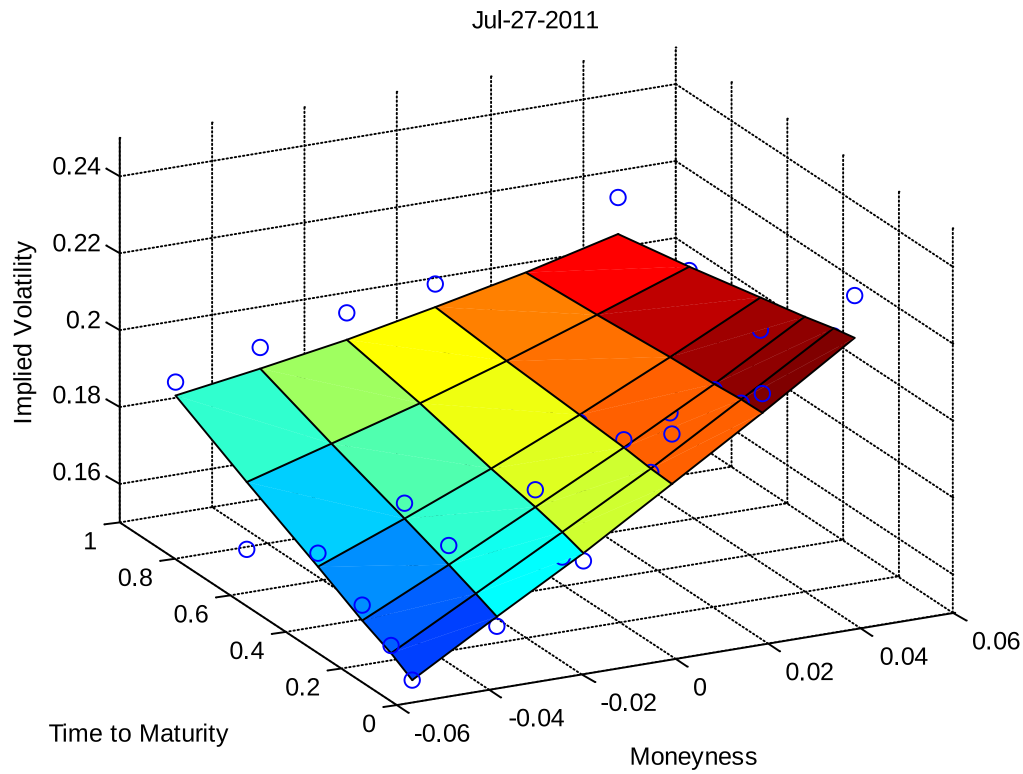
Appendix

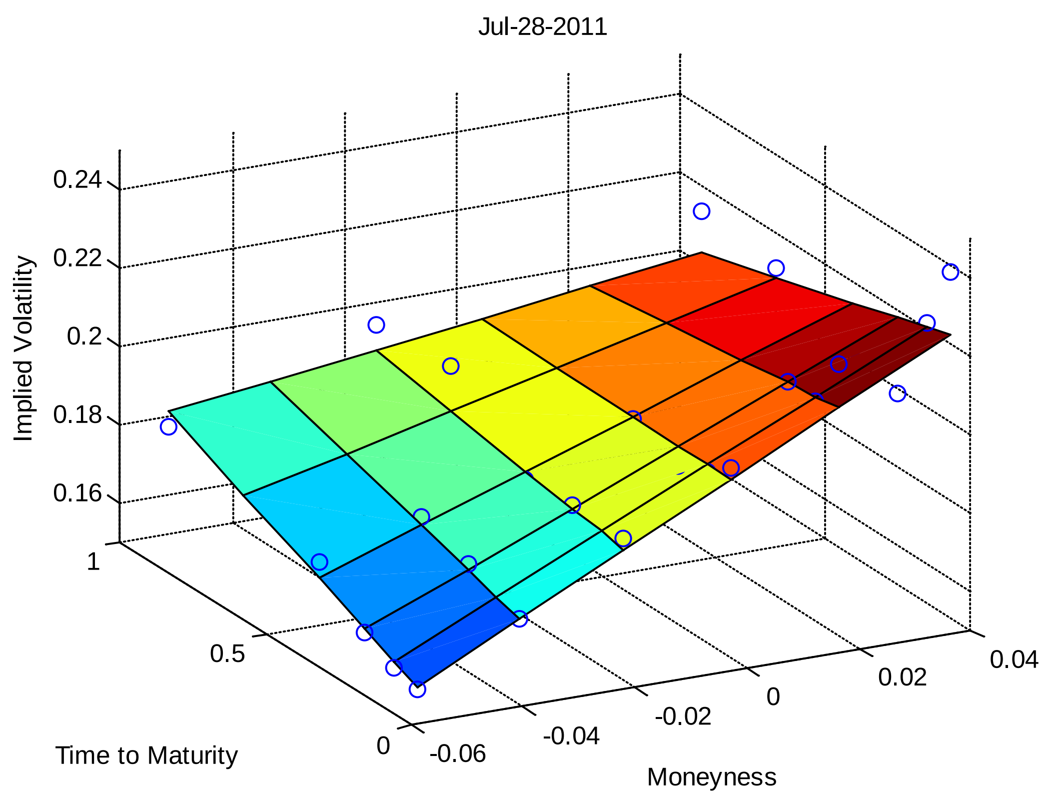


Jul-25-2011
0.24
0.22
0.2
0.18
0.16
Implied Volatility
1
0.5
0
Time to Maturity
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Moneyness



Jul-26-2011
0.24
0.22
0.2
0.18
0.16
0.14
0.12
Implied Volatility
1
0.5
0
Time to Maturity
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Moneyness


Jul-27-2011
Implied Volatility
0.24
0.22
0.2
0.18
0.16
1
0.8
0.6
0.4
0.2
0
Time to Maturity
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
Moneyness


Jul-28-2011
Implied Volatility
0.24
0.22
0.2
0.18
0.16
1
0.5
0
Time to Maturity
-0.06
-0.04
-0.02
0
0.02
0.04
Moneyness